

# Hole Deviation is Defined

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## Abstract

What is hole deviation? Why and how should hole deviation be quantified? What can merely a directional survey and a plan really tell us about deviation control? The answers to these questions are addressed in this paper. In addition, with the mechanisms presented herein, a directional driller can make more-informed deviation control decisions or an automated directional drilling system can be made “smarter”, and directional control performance can be monitored in real time.

This paper contains new formulae, which mathematically define hole deviation. Collectively, eight components define hole deviation. Understanding hole deviation is required for a complete working knowledge of spatial differences between the actual drill hole and the planned path of the wellbore. The components that define hole deviation are based on lineal and angular differences—and the relative changes thereof—between the actual drill hole and the relevant nearest point on the planned path. Survey data and a mathematically-defined planned path of the wellbore, comprise the necessary information to compute hole deviation. A method is detailed that finds the minimum distance between a survey station and the planned path, and the corresponding planned measured depth. A hole deviation log is presented for a well drilled in China. Hole deviation, as defined herein, is discussed. Finally, a new type of well design, which is entirely practical given the advent of steerable rotary directional drilling tools, is introduced to address transitions between linear and curved hole sections.

## Introduction

Drilling directionally began in the mining industry in the early 1900's<sup>1-4</sup>. At least a decade passed before the petroleum industry initiated similar “directed drilling” techniques<sup>5</sup>. The 1930's saw oil wells and blowout-relief wells drilled directionally<sup>6,7</sup>. For many years thereafter, the art of directional drilling remained as a very specialized drilling technique within the petroleum industry. Directional drilling technology went global after the development of marine platforms during the 1960's<sup>8</sup>.

Results from much research have been published during the latter half of the twentieth century regarding the mechanisms that affect the direction in which a bit drills. Technical bottomhole assembly (BHA) modeling began with the published works of Arthur Lubinski<sup>9</sup>. From then onward, even a partial list of contributors to the subjects hole deviation, deviation tendencies, drilling trajectory prediction, directional drilling modeling, and BHA design, is lengthy<sup>10-54</sup>.

To the best of the author's knowledge, a mathematical definition of hole deviation has never been published. Given the abundance of work directed at understanding hole deviation, this observation is ironic. Perhaps a “rigorous” definition of hole deviation is unnecessary since the viewer of directional plots can mentally “determine” hole deviation. Perhaps it is a burden left to the expertise of the directional driller. Or, possibly the subject is far more complicated than we would like to admit.

## Importance of Technical Hole Deviation

Technical hole deviation describes geometric differences between actual and planned drill paths. A fundamental factor that affects deviation control response is interpretation of hole deviation. Traditionally, a directional driller “mentally” interprets hole deviation and then concludes deviation control response (tool setting, etc.).

A quantitative description of hole deviation is useful for three primary reasons. First, providing the directional driller with a “hole deviation log” may improve deviation control performance by influencing his deviation control response, especially when drilling three-dimensionally-planned well paths where mental inference of hole deviation is often considerably complicated. Second, closed-loop (also known as “automated”) directional drilling systems are machines that require numeric values to determine control output (i.e., tool settings); obvious system “inputs” are metrics that address hole deviation. Third, hole deviation—as later defined herein—contains much more information about directional drilling performance than is conveyed in directional vertical section and plan view plots.

A hole deviation log, the details of which are discussed later in this paper, succinctly conveys hole deviation. It equips the operator with a superior mechanism with which to monitor the progress of directional control as a well is drilled. It also provides the foundation of a means to compare overall directional control performance across multiple wells and/or service companies. A hole deviation log can be generated as each new survey station is acquired, or at any time thereafter.

## Mathematically-Defined Planned Path

Respective to hole deviation, a preferable method by which to mathematically represent the entire planned drill path is to parametrically define each Cartesian coordinate, and hole inclination and azimuth, in terms of measured depth

(MD). That is, the planned path is designed, and then represented as follows.

$$N(MD) = P_1(MD) \quad [\text{Eq. 1}]$$

$$E(MD) = P_2(MD) \quad [\text{Eq. 2}]$$

$$TVD(MD) = P_3(MD) \quad [\text{Eq. 3}]$$

$$f(MD) = P_4(MD) \quad [\text{Eq. 4}]$$

$$q(MD) = P_5(MD) \quad [\text{Eq. 5}]$$

$N$ ,  $E$ , and  $TVD$  represent earth-fixed Cartesian coordinates North, East, and True Vertical Depth, respectively, and  $P_i$  are applicable functions dependent on the well path design.  $f$  and  $q$  represent planned hole inclination and azimuth, respectively.  $MD$  ranges from zero to planned total depth.

### Minimum Distance between Hole Bottom and Plan

As the hole is drilled, it is necessary to determine where on the plan one would prefer the wellbore to exist. The linear distance between the current bottomhole location ( $N_b$ ,  $E_b$ ,  $TVD_b$ ) and a point on the planned path is computed with the

three-dimensional distance formula. This is generally represented by [Eq.6].

$$D3D(N_b, E_b, TVD_b, MD) = \sqrt{(N_b - N(MD))^2 + (E_b - E(MD))^2 + (TVD_b - TVD(MD))^2} \quad [\text{Eq. 6}]$$

Let  $MD^*$  represent the measured depth along the planned path, whose respective Cartesian coordinates ( $N^*$ ,  $E^*$ ,  $TVD^*$ ) = ( $P_1(MD^*)$ ,  $P_2(MD^*)$ ,  $P_3(MD^*)$ ) minimize the distance computed with [Eq. 6].  $MD^*$  is found by taking the derivative of [Eq. 6] with respect to  $MD$  and setting the result equal to zero. Thus, parametric functions, which define the derivatives of the planned earth-fixed Cartesian coordinates, are also needed. [Eq. 7] is the derivative of [Eq. 6] with constants omitted.

The measured depth that sets the right hand side of [Eq. 7] equal to zero is  $MD^*$ . The numerator of the right hand side of [Eq. 7] makes the only relevant contribution to finding a practical root. Therefore, the denominator may be ignored and  $MD^*$  is found by solving [Eq. 8].

For practical purposes, [Eq. 8] is a piecewise-continuous, *monotonically increasing* function. Thus, finding  $MD^*$  with [Eq. 8] is a simple numerical task, and a logical initial guess is a value less than the current total depth.

$$\frac{dD3D}{dMD}(MD) = \frac{(N(MD) - N_b) \frac{dN(MD)}{dMD} + (E(MD) - E_b) \frac{dE(MD)}{dMD} + (TVD(MD) - TVD_b) \frac{dTVD(MD)}{dMD}}{\sqrt{(N_b - N(MD))^2 + (E_b - E(MD))^2 + (TVD_b - TVD(MD))^2}} \quad [\text{Eq. 7}]$$

$$dMD3D(MD) = (N(MD) - N_b) \frac{dN(MD)}{dMD} + (E(MD) - E_b) \frac{dE(MD)}{dMD} + (TVD(MD) - TVD_b) \frac{dTVD(MD)}{dMD} \quad [\text{Eq. 8}]$$

$P_i$  and  $dP_i/dMD$  are piecewise-continuous functions. The functional form of each  $P_i$  component depends on whether the respective interval is linear or curved. Component functions of  $P_i$  for a 3D circular hole section (e.g., simultaneous build and right turn) may be acquired via any general 3D circular-arc well planning method that includes interpolation formulae<sup>55</sup>. Minimum-curvature survey calculational methods that include interpolation formulae<sup>56,57</sup> may also be employed to determine component functions of  $P_i$  for 3D circular hole sections; either route leads to identical numeric solutions. The component functions of  $P_i$  for a linear hole section are determined with the cosines of the line.

Planned drill paths comprised of linear and circular hole sections can always be defined as proposed in [Eqs. 1-5], and their derivatives determined symbolically. When  $MD^*$  has been found, the task of computing the associated coordinates and angles is straightforward with the use of [Eqs. 1-5]. Let the following variables be defined.

- $N^*$  = North coordinate of planned path at  $MD^*$ ; L
- $E^*$  = East coordinate of planned path at  $MD^*$ ; L
- $TVD^*$  = True Vertical Depth coordinate of planned path at  $MD^*$ ; L
- $f^*$  = inclination of planned path at  $MD^*$ ; degrees
- $q^*$  = azimuth of planned path at  $MD^*$ ; degrees
- $N_b$  = North coordinate of current bottomhole location; L
- $E_b$  = East coordinate of current bottomhole location; L

$TVD_b$  = True Vertical Depth coordinate of current bottomhole location; L

$f_b$  = inclination at current bottomhole location; degrees

$q_b$  = azimuth at current bottomhole location; degrees

### Definition of Hole Deviation

As was stated, technical hole deviation describes geometric discrepancies between actual and planned drill paths. Hole deviation is a matter of definition, as opposed to derivation. The following definition of hole deviation originated from work performed by the author in which a control algorithm was developed for closed-loop directional drilling, given a mathematically defined planned path. While the patent-pending, foregoing control methodology is applicable to steerable rotary directional drilling systems, the input (i.e., hole deviation) is applicable to any type of directional drilling.

Eight variables are necessary to sufficiently quantify hole deviation. They are listed below and then mathematically defined.

- $V$  = vertical deviation; L
- $H$  = horizontal deviation; L
- $Df$  = inclinational deviation; degrees
- $Dq$  = azimuthal deviation; degrees

$DV_r$  = relative change in vertical deviation; L/L  
 $DH_r$  = relative change in horizontal deviation; L/L  
 $DDf_r$  = relative change in inclinational deviation; degrees/L  
 $DDq_r$  = relative change in azimuthal deviation; degrees/L

$$V = \cos(\mathbf{q}^*)\cos(\mathbf{f}^*)(N_b - N^*) + \sin(\mathbf{q}^*)\cos(\mathbf{f}^*)(E_b - E^*) - \sin(\mathbf{f}^*)(TVD_b - TVD^*) \quad [\text{Eq. 9}]$$

$$H = \cos(\mathbf{q}^*)(E_b - E^*) - \sin(\mathbf{q}^*)(N_b - N^*) \quad [\text{Eq. 10}]$$

$$Df = f_b - f^* \quad [\text{Eq. 11}]$$

$$Dq = q_b - q^* \quad [\text{Eq. 12}]$$

$$\Delta V_r^n = 1000 \frac{V^n - V^{n-1}}{\Delta L^n} \quad [\text{Eq. 13}]$$

$$\Delta H_r^n = 1000 \frac{H^n - H^{n-1}}{\Delta L^n} \quad [\text{Eq. 14}]$$

$$\Delta \Delta f_r^n = 100 \frac{\Delta f^n - \Delta f^{n-1}}{\Delta L^n} \quad [\text{Eq. 15}]$$

$$\Delta \Delta q_r^n = 100 \frac{\Delta q^n - \Delta q^{n-1}}{\Delta L^n} \quad [\text{Eq. 16}]$$

The superscript "n" in the definitions of each "relative change in ..." refers to the respective values during the current computing of hole deviation; "n-1" refers to values at the prior computing. The term  $DL$  refers to the length of planned hole "drilled" between the two foregoing hole deviation computations. Thus,  $DL$  is  $MD^{*(n)} - MD^{*(n-1)}$ .  $DL$  is preferably somewhat short; for example, 10-90 feet (3-27 meters). The constants (1000 and 100) are included solely for convenience when plotting.

### Fundamentals of Hole Deviation as Defined Herein

Hole deviation is defined with properties of the nearest point on the planned path (i.e., at  $MD^*$ ). Vertical and horizontal deviations are lineal differences. Inclinational and azimuthal deviations are angular differences. These first-order differences represent current states. The relative changes in vertical, horizontal, inclinational, and azimuthal deviations are second-order differences that measure how the respective state is changing as the hole is drilled.

1st order	$V$	} lineal {	$H$	} angular {	$Df$	} angular {	$Dq$
2nd order	$DV_r$		$DH_r$		$DDf_r$		$DDq_r$

Vertical ( $V$ ) and horizontal ( $H$ ) deviations may be viewed in the mind's eye as looking down the planned hole from  $MD^*$  in an orientation parallel to a line defined by  $f^*$  and  $q^*$ , and seeing the current hole bottom in the periphery. From this viewpoint,  $+V$  points to the high side of the hole;  $+H$  points to the right side of the hole.

$V$  and  $H$  are components of a vector, which points from  $(N^*, E^*, TVD^*)$  to  $(N_b, E_b, TVD_b)$ .  $V$  and  $H$  are portrayed in Figure 1. Performing two successive coordinate axis rotations derive the equations for  $V$  and  $H$ . The first rotation is by  $q^*$  about the  $TVD$  axis. The second rotation is by  $f^*$  about the  $E'$  axis. By definition, the aforementioned vector is orthogonal to the planned path at  $MD^*$ . As such, a numerical check to insure  $MD^*$  found via [Eq. 8] is correct, is the requirement  $\Delta TVD''$  equals zero; i.e.,

$$\Delta TVD'' = \sin(\phi^*)\cos(\theta^*)(N_b - N^*) + \sin(\phi^*)\sin(\theta^*)(E_b - E^*) + \cos(\phi^*)(TVD_b - TVD^*) = 0 \quad [\text{Eq. 17}]$$

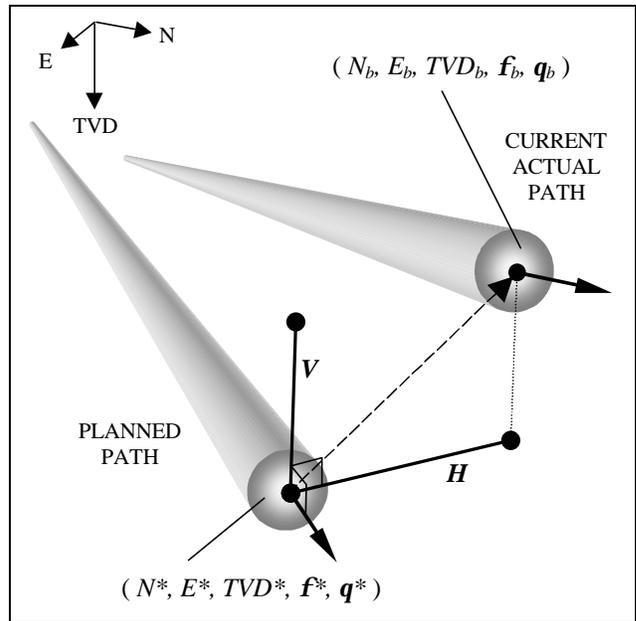


Figure 1: Sketch depicting "high" vertical deviation ( $V$ ) and "left" horizontal deviation ( $H$ ).

Inclinational ( $Df$ ) and azimuthal ( $Dq$ ) deviations are more difficult to visualize than  $V$  and  $H$ .  $Df$  and  $Dq$  are differences in wellbore angles. The *relative changes* in vertical ( $DV_r$ ), horizontal ( $DH_r$ ), inclinational ( $DDf_r$ ), and azimuthal ( $DDq_r$ ) deviations are far less "intuitive" than are  $V$ ,  $H$ ,  $Df$ , and  $Dq$ ; however, they are *extremely* informative. An analogous example to convey their importance might be the significance of determining a car's position *and* velocity before deciding whether to cross a street. Only if all eight hole deviation components equal zero does an actual drill path perfectly follow a planned drilling trajectory over  $DL$ .

$V$  and  $H$  are lengths (feet, meters).  $Df$  and  $Dq$  are angles (degrees).  $DV_r$  and  $DH_r$  are dimensionless numbers. While  $DDf_r$  and  $DDq_r$  have units similar to dogleg-severity, they are not measures of wellbore curvature. Indeed, borehole dogleg-severity could equal 10 degrees per 100 feet while  $DDf_r$  could be nil, and vice versa.

Most often, simultaneous interpretation of eight variables is not a simple task. Fortunately, it is possible and logical to segregate hole deviation into its "vertical" and "horizontal" constituents. This observation transforms the number of variables to interpret into two groups of four. "Vertical"

constituents include  $V$ ,  $DF$ ,  $DV_r$ , and  $DDF_r$ ; “horizontal” constituents include  $H$ ,  $Dq$ ,  $DH_r$ , and  $DDq_r$ .

### Hole Deviation Log – China Well

Table 1 presents the critical points of a sidetrack directional well drilled in China. Measured depths and coordinates are stated relative to a window that was cut at 5050 ft (MD). As Table 1 shows, the directional plan, beginning with an inclination and azimuth of 56.2 and 84.3 degrees, respectively, was to drop inclination while turning right to an azimuth of 203 degrees. A tangent section was to follow the preceding 3 degrees per 100 ft drop/right-turn section. After holding angle for about 2300 ft, hole inclination was to build while turning back left, to hit the target drilling horizontally and heading due south. The build/left-turn section was to be drilled at 6 degrees per 100 ft. Standard plan and vertical section views are presented in Figure 2 and Figure 3, respectively. Directional drilling was conducted with a bent-housing mud motor. The computations of hole deviation were not available when the well was drilled.

Hole Section	Description	MD* ft	N* ft	E* ft	TVD* ft	Inc.* deg	Azi.* deg	DLS* deg/100ft
0	Begin Drop & Right Turn	0	0	0	0	56.19	84.34	-----
1	End Drop & Right Turn	2849	-1106	941	2137	49.82	202.91	3.048
2	End Hold	5167	-2737	252	3633	49.82	202.91	0
3	End Build & Left Turn	5901	-3397	137	3886	89.43	180	6.096
4	End Horizontal	7542	-5037	137	3902	89.43	180	0

Table 1: 3D sidetrack directional well plan.

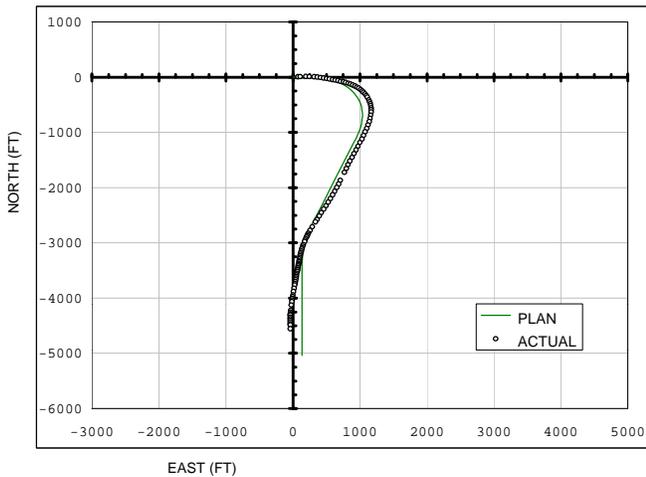


Figure 2: Plan view of the example well.

Less-typical plots are presented in Figures 4-6. The graph of TVD versus  $MD^*$  (Figure 4) portrays much better (and more realistic) true vertical depth control than does the vertical section view (Figure 3). While Figures 5 and 6 are informative and self-explanatory to the trained reader, a more efficient means is needed to collectively convey the geometric aspects of directional performance; hence, the Hole Deviation Log.

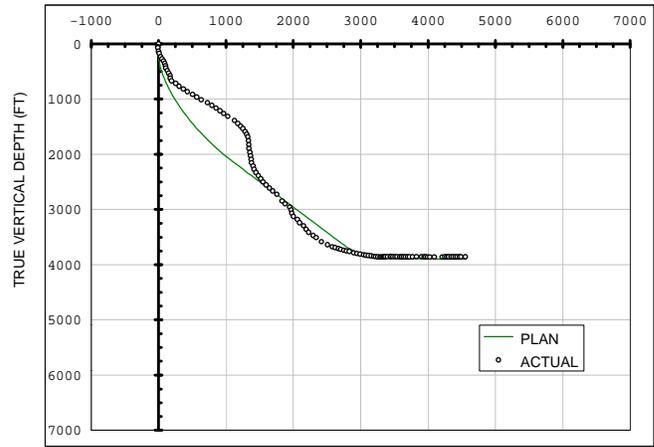


Figure 3: Vertical Section view of the example well.

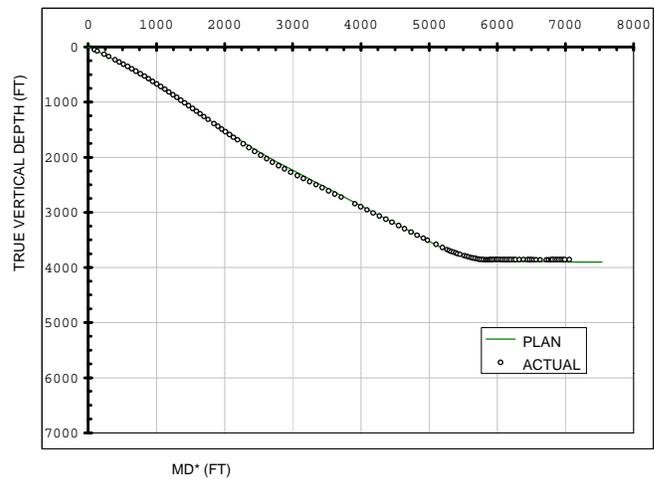


Figure 4: TVD versus  $MD^*$  of the example well.

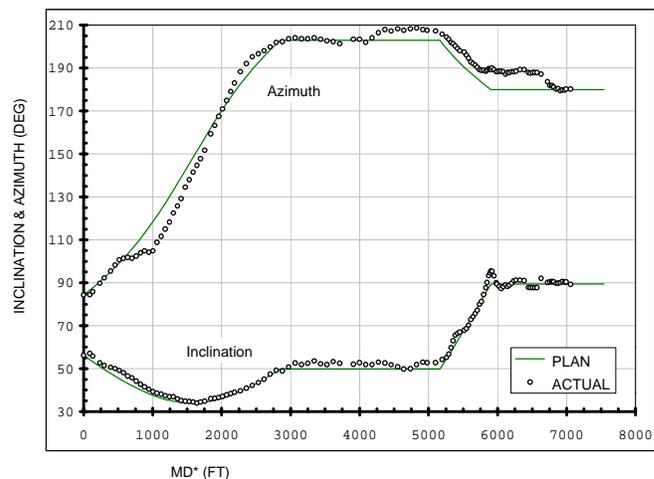


Figure 5: Inclination and azimuth profiles of the example well.

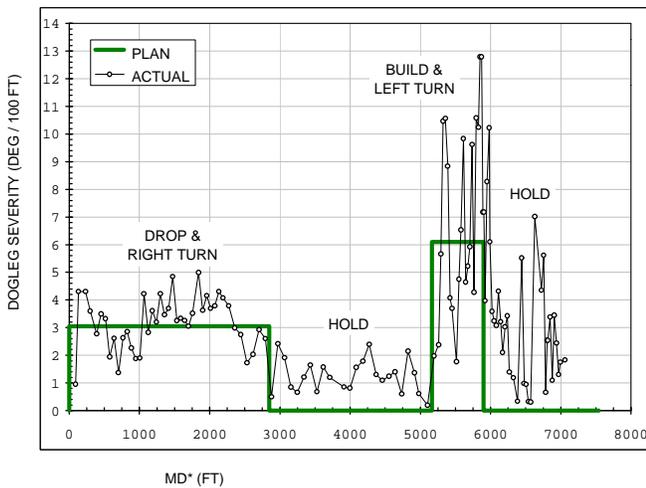


Figure 6: Dogleg-severity profile of the example well.

A hole deviation log should concisely display the directional well plan, and deviations from the Directional well plan. Accordingly, consider the Hole Deviation Logs presented in Figure 7 and Figure 8 for the example well. All values are plotted versus MD. An asterisk denotes a planned value.

Figure 7 is a “vertical” Hole Deviation Log. Four tracks are displayed. The outer left track graphs planned and actual wellbore inclination. Planned and actual dogleg-severity (DLS) is graphed on the inner left track. The circular unfilled markers on the actual DLS curve are present to convey depths with survey stations. The circular filled markers on the planned DLS curve are present to convey critical-point changes in the well plan (e.g., curved to straight).

Hole deviation in the vertical sense is displayed on the two right tracks of Figure 7. The inner right track displays vertical deviation ( $msVD^{TM}$ , same as  $V$ ) and the relative change in vertical deviation ( $RCVD^{TM}$ , same as  $DV_r$ ). The outer right track displays inclinational deviation ( $msID^{TM}$ , same as  $Df$ ) and the relative change in inclinational deviation ( $RCID^{TM}$ , same as  $DDf_r$ ). Zero centers each of the two right tracks. As previously stated, for the actual drill path to follow the planned path in the vertical sense,  $msVD$ ,  $RCVD$ ,  $msID$ , and  $RCID$  must trace their respective zero lines.

Figure 8 is a “horizontal” Hole Deviation Log. Four tracks are displayed. The outer left track graphs planned and actual wellbore azimuth. Again, planned and actual dogleg-severity (DLS) is graphed on the inner left track.

Hole deviation in the horizontal sense is displayed on the two right tracks of Figure 7. The inner right track displays horizontal deviation ( $msHD^{TM}$ , same as  $H$ ) and the relative change in horizontal deviation ( $RCHD^{TM}$ , same as  $DH_r$ ). The outer right track displays azimuthal deviation ( $msAD^{TM}$ , same as  $Dq$ ) and the relative change in azimuthal deviation ( $RCAD^{TM}$ , same as  $DDq_r$ ). Zero centers each of the two right tracks. For the actual drill path to follow the planned path in the horizontal sense,  $msHD$ ,  $RCHD$ ,  $msAD$ , and  $RCAD$  must trace their respective zero lines.

Numerical values of hole deviation for the example well are given in Table 2.

### Hole Deviation – Foundation of Directional Control

Let us wind-back the clock and imagine the bit drilling new hole at 1000 ft MD for the example well. The vertical hole deviation log shows the wellbore is 40 ft high ( $msVD \cong 40$ ) of the plan. Hole inclination is a little high but steadily approaching the plan ( $msID$  positive,  $RCID$  slightly negative). The relative change in vertical deviation is zero, but quickly heading negative ( $RCVD$  0→neg), thus, we know the hole will soon head back towards the plan. The tangent section doesn’t begin for another 2000 ft. What do you do? How should you alter the directional tool settings, if at all?

Inference of hole deviation—however it is defined—when combined with expectancy of system response in relation to the remaining planned path, dictates directional control actions made while drilling. For this reason, a set of rules may be compiled and employed to map hole deviation into a directional control action, in effort to drill close to the planned trajectory. In other words, it is possible for a directional driller (or an algorithm) to interpret/process values contained in the hole deviation log, and determine the next adjustment to a directional tool, in order to better track the plan. The details, which defend the foregoing statements, are patent-pending<sup>58</sup> and are postponed to a later paper.

ACTUAL							NEAREST POINT ON PLAN						HOLE DEVIATION							
MD	N	E	TVD	Inc	Azi	DLS	MD*	N*	E*	TVD*	Inc*	Azi*	V	H	Df	Dq	DV <sub>r</sub>	DH <sub>r</sub>	DDf <sub>r</sub>	DDq <sub>r</sub>
0.00	0.00	0.00	0.00	56.19	84.34	-----	0.00	0.00	0.00	0.00	56.19	84.34	0.00	0.00	0.00	0.00	-----	-----	-----	-----
90.88	7.41	75.52	50.00	57.06	84.42	0.96	90.78	5.88	74.28	51.84	54.16	86.63	2.28	-1.46	2.90	-2.21	-----	-----	-----	-----
131.23	10.27	109.06	72.31	55.81	85.87	4.31	131.02	7.49	106.67	75.65	53.28	87.69	4.17	-2.68	2.53	-1.82	46.97	-30.32	-0.92	0.97
234.58	13.48	192.81	132.71	52.70	89.80	4.31	233.89	8.81	187.89	138.75	51.06	90.50	7.77	-4.72	1.64	-0.70	35.00	-19.83	-0.87	1.09
300.20	12.57	244.55	173.06	51.40	92.30	3.60	299.15	7.56	238.14	180.37	49.69	92.37	9.59	-5.27	1.71	-0.07	27.89	-8.43	0.11	0.97
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7108.92	-4464.90	-36.98	3857.84	90.30	179.80	1.30	6968.92	-4464.51	136.71	3896.71	89.43	180.00	38.87	173.69	0.87	-0.20	19.53	-4.26	-1.30	0.00
7132.55	-4488.52	-36.98	3857.71	90.40	180.20	1.75	6992.54	-4488.13	136.71	3896.95	89.43	180.00	39.24	173.69	0.97	0.20	15.66	0.00	0.42	1.69
7198.16	-4554.13	-37.20	3857.94	89.20	180.20	1.83	7058.16	-4553.74	136.71	3897.60	89.43	180.00	39.66	173.92	-0.23	0.20	6.40	3.51	-1.83	0.00

Table 2: Values of Hole Deviation for the example well. An asterisk denotes a planned value. The original directional survey was reported in meters and was converted to feet; hence, the extra digits on actual measured depth.

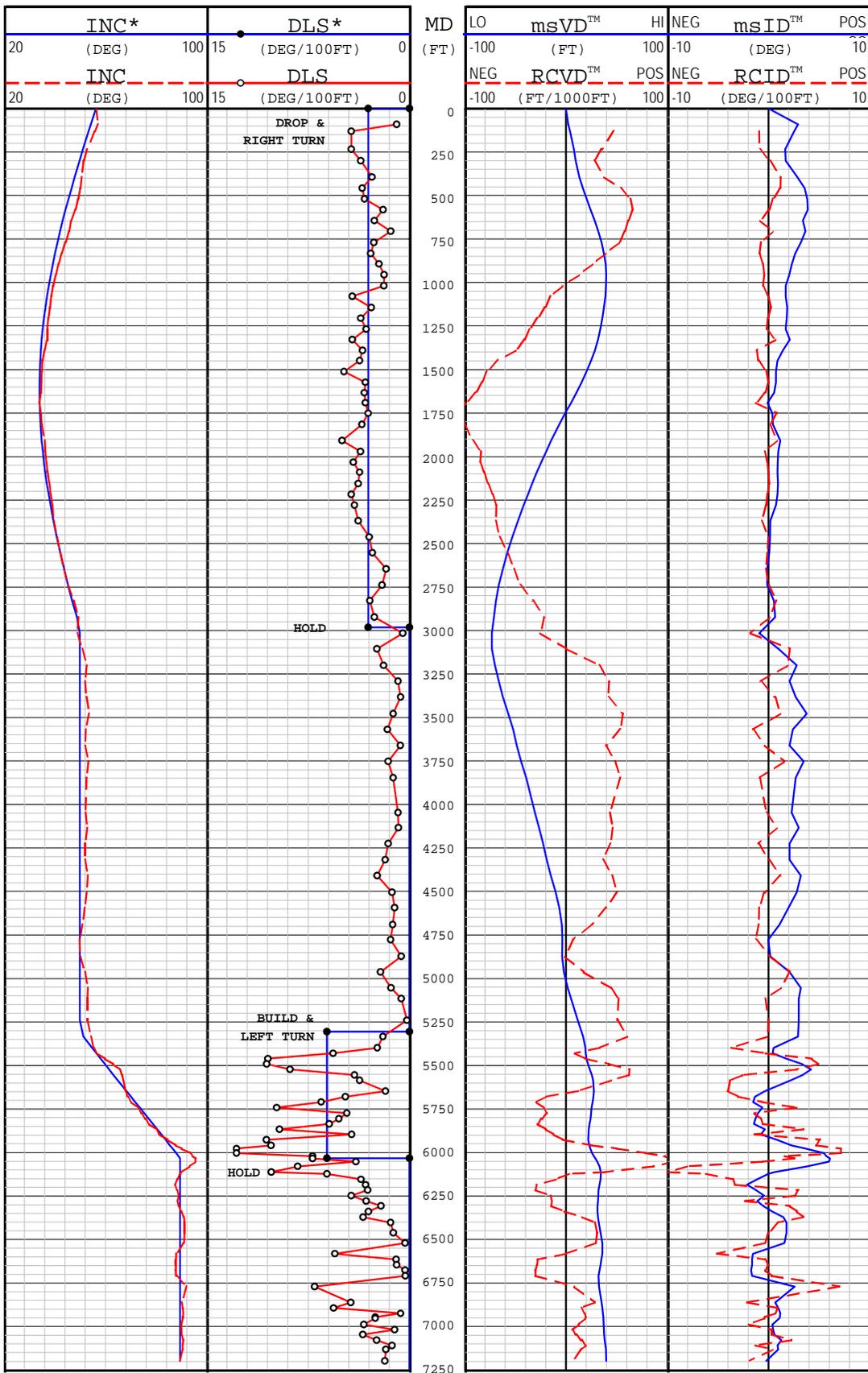


Figure 7: Vertical Hole Deviation Log of the example well. An asterisk denotes a planned value.

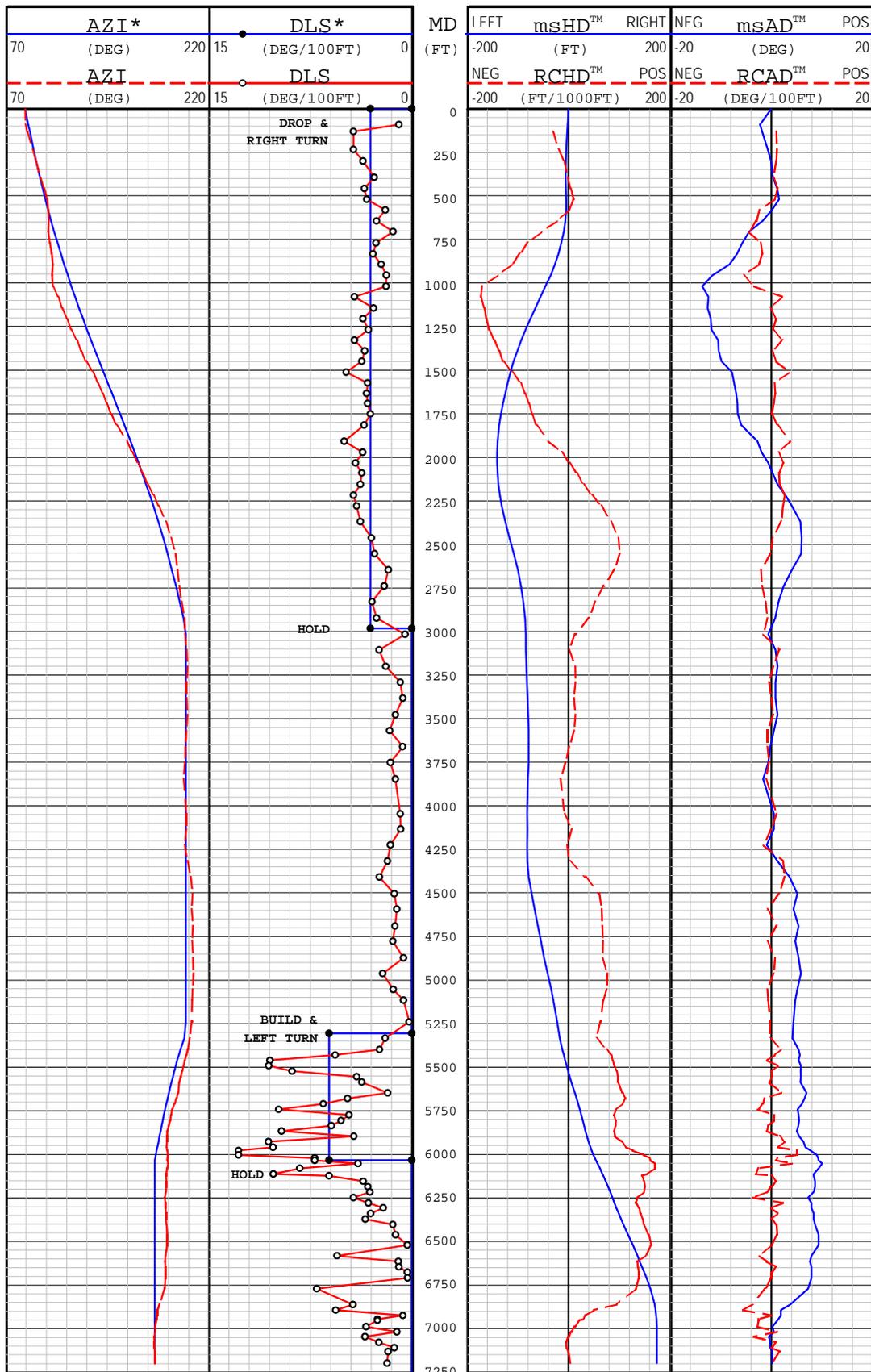


Figure 8: Horizontal Hole Deviation Log of the example well. An asterisk denotes a planned value.

The task of a directional driller, or that of an automated directional drilling control system, requires “tuning” eight dimensions. This observation helps to explain the complexity of directional drilling trajectory control. Because directional drillers are humans, directional “performance” can easily vary for a variety of reasons. Steps toward automated or partially-automated control systems should alleviate performance variability.

### Future Planned Well Paths

Planned directional drill paths are comprised of a finite number of connecting linear and curved hole sections. The most common curved section is a segment of a circle. At least one other curved drilling trajectory exists, i.e., the catenary method<sup>59</sup>.

As new downhole-adjustable directional drilling tools<sup>60</sup> become available to industry, planned non-linear drill paths will expand beyond constant-curvature (i.e., circular) hole sections. When considering the importance of “smooth” boreholes, especially in extended reach drilling applications, it is advantageous for the planned hole-inclination and hole-azimuth profiles to be smooth (i.e., continuously differentiable) with respect to measured depth. In other words, planned dogleg-severity (DLS) should change gradually at hole-section transitions. Drilling a circular hole section in sequence with a linear hole section—the current industry standard—creates an abrupt change in DLS near the transition.

A planned hole-inclination profile for a 2D horizontal well is presented in Figure 9. For the traditional case comprising a constant build gradient, the kick-off point is  $K_1$ , and horizontal is reached at the measured depth associated with  $H$ . As Lubinski<sup>17</sup> stated in the 1960’s, dogleg-severity measures the change in “overall angle” of the wellbore. Thus, an abrupt change in DLS exists at  $K_1$  and at  $H$  for the traditional case (e.g., 3 degrees per 100 ft, instantaneously, to 0 degrees per 100 ft, even though the instantaneous change in overall angle of 90 degrees is zero).

For the non-traditional case of employing a variable build gradient with target constraints held constant, the kick-off point is at  $K_2$ , which is more shallow than  $K_1$ . With the construct presented in Figure 9, the change in DLS is gradual between the linear and curved hole section transitions. That is, the acceleration or change in the change of overall angle is gradual; not abrupt. The drill path between  $A$  and  $B$  is of the same shape as that between  $K_1$  and  $H$ , i.e., circular. The drill path between  $K_2$  and  $A$ , for example, could follow a cubic equation.

An implication of this well design is a more-controllable trajectory at a hole section transition, because the inherent follow-through characteristics of a bottomhole assembly, associated with a change in build gradient, would be lessened. (Observe a typical inclination overshoot at a measured depth around 6000 ft in Figure 7. Transients of this nature cause excessive DLS, and thus, burden hole quality for the remaining life of the well.) Another likely result is less overall torque-and-drag while drilling anywhere below  $K_2$ . Steerable rotary directional drilling tools are well suited to the proposed type of well design, because the direction and magnitude of bit forces can be altered with downhole tool-setting adjustments.

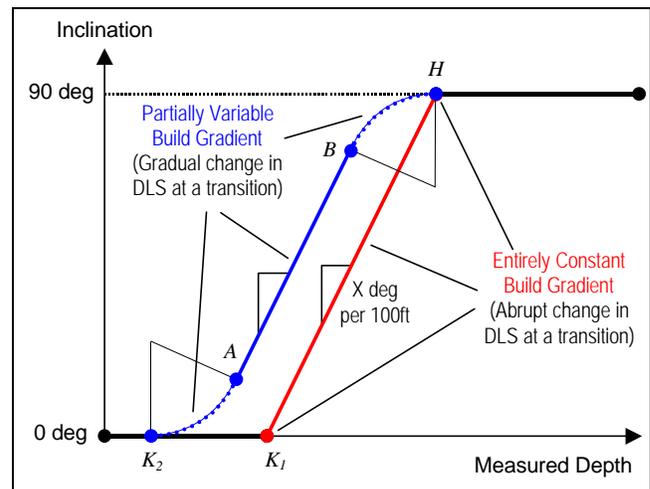


Figure 9: Planned hole-inclination profile for a 2D horizontal well: traditional constant build gradient, and proposed variable build gradient (now practical with steerable rotary directional drilling tools).

### Conclusions

1. Hole deviation has been mathematically defined.
2. The requirements to calculate hole deviation include directional survey data and a mathematically defined planned path (preferably in parametric form).
3. A hole deviation log has been devised to concisely display the directional well plan and deviations from the directional well plan.
4. Inference of hole deviation provides the foundation upon which directional control actions are made.
5. Operators and directional drilling companies should investigate the merits of directional well designs that incorporate gradual changes in DLS at transitions between linear and curved hole sections.

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